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# Walking or running in the rain—a simple derivation of a general solution

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**Abstract**

The question whether to walk slowly or to run when it starts raining in order to stay as dry as possible has been considered for many years—and with different results, depending on the assumptions made and the mathematical descriptions for the situation. Because of the practical meaning for real life and the inconsistent results depending on the chosen parameters, this problem is well suited to undergraduate students learning to decide which parameters are important and choosing reasonable values to describe a physical problem. Dealing with physical parameters is still useful at university level, as students do not always recognize the connection between pure numbers and their qualitative and quantitative influence on a physical problem. This paper presents an intuitive approach which offers the additional advantage of being more detailed, allowing for more parameters to be tested than the simple models proposed in most other publications.

## 1. Introduction

Imagine leaving a building, noting that it has started to rain and you do not have an umbrella. Is it better to walk at usual velocity or to run at maximum speed, or can the best way to keep as dry as possible be found somewhere between these extreme values? Which parameters have to be taken into consideration for the best possible decision?

Several publications mention this problem. Some of them deal only with the simple case that the rain falls down vertically, leading to the conclusion that running as fast as possible is the best choice [1]. Other authors take the speed of the wind into consideration [2].[3] These articles conclude that with the wind on the back it is best to walk at the same speed as that of the wind. De Angelis gives an equation for the influence of wind and the conditions for

[3] The problem is also discussed in Adams C 1992 Which will keep you drier, running through the rain or walking? http://www.straightdope.com/columns/read/827/which-will-keep-you-drier-running-through-the-rain-or-walking; Craigen D, Is it worth running in the rain? http://www.dctech.com/physics/features/0500.php.

  

an optimum walking velocity without derivation. For purely vertical rain, he calculated that a person sprinting very fast gets 10% less wet than someone walking hastily [2].

In addition to these theoretical investigations, practical experiments have been performed. In the TV show 'MythBusters' the tests lead to the result that a running person becomes even wetter than someone walking.[4] Peterson and Wallis travelled with different velocities in a rainstorm and used the weight of their soaked clothes to determine the amount of rain which they collected along a defined way. They found that running leads to about 40% less raindrops in the garment than walking, confirming their calculation that running quickly is always best [3].

Some articles citing the latter experiment[5] and several websites discussing the problem[6] show the general interest in this task—obviously because it belongs to 'real life' and is thus more than a pure academic mind game. However, even if the above-mentioned approaches do take into account the influence of the horizontal velocity component of the rain, none of them examines the influence of the shape of the human body.

As Bailey has shown for a cuboid [4], the optimal choice of the velocity is not independent of the dimensions of this body. He concludes—taking into consideration rain from the side—that a heavier person should always run as fast as possible, while a slimmer person should walk with the horizontal speed of the rain if it comes from the back.

Our paper is targeted at developing an easier approach than the latter, using an intuitive ansatz which, however, allows for a broad variety of parameters whose influence can easily be tested by the students. While more experienced students should be able to develop an approach to the solution of this problem themselves, undergraduates in the first semester can be supported by the teacher in this derivation and afterwards 'play' with the different parameters. In this way, students can get a feeling for reasonable values of variables and also for the importance of parameters—while some effects may be negligible, others can strongly influence the result. Moreover, such a physical problem taken from real life offers the opportunity to check whether the results are likely. This is something students often forget while solving physical problems.

The paper is structured as follows. In section 2, a simple mathematical model is presented. Section 3 shows some results, calculated by means of the model, and gives a general overview of the influence of different parameters on the solution of the problem described above. In section 4, several continuative tasks and ideas are described, which allow the students to elaborate on the first solution further, taking into account more parameters, and to test to what extent different variables influence the results. Section 5 gives a conclusion of the results of the paper.

## 2. Mathematical model

For the following calculation, some assumptions have to be made.

In contrast to other models, we regard the person in the rain as a cylinder with height $h$ and radius $r$, instead of the cuboid usually examined. The local coordinate system is assumed to move with the person in order to support the correlation with the respective real-life situation.

[4] 'MythBusters', Episode 1, Discovery Channel 23 September 2003.

[5] ZEIT ONLINE, section 'Stimmt's?', repeating the main results of [2], http://www.zeit.de/stimmts/1998/1998_21_stimmts; additional to a summary of the results of [2], a calculation of rainstorms 'without boundary in space' is available at http://everything2.com/title/Is%2520it%2520better%2520to%2520run%2520or%2520walk%2520in%2520the%2520rain%253F; *The New York Times* online article 'Really? The claim: walking in the rain keeps you drier than running', http://www.nytimes.com/2006/10/24/health/24real.html/partner/rssnyt/.

[6] Question and answers on PhysLink.com http://www.physlink.com/Education/AskExperts/ae212.cfm.

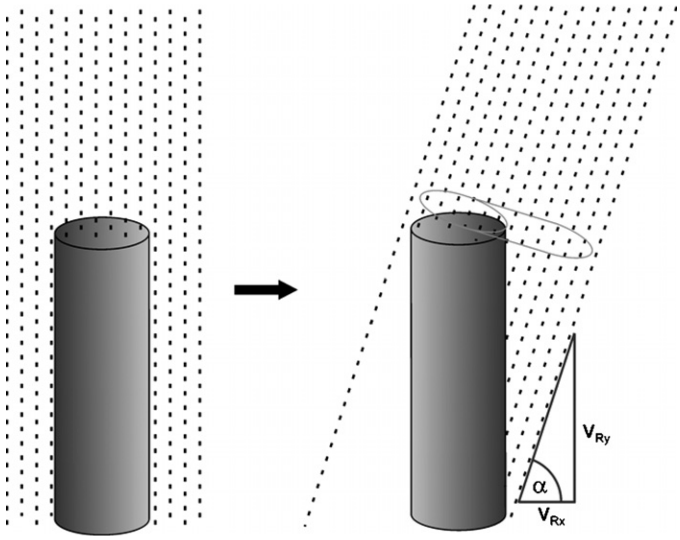


**Figure 1.** Schematic depiction of rain from above (left) and rain with relative horizontal velocity component (right).

If the person walks or runs with a velocity $v_x$ and the rain comes directly from above, the rain has a horizontal velocity component $v_{Rx} = -v_x$ in the coordinate system moving with this person. For rain which has an additional horizontal component $v_H$ in the rest frame, the total horizontal velocity component in the moving coordinate system is $v_{Rx} = -v_x + v_H$. Thus, $v_{Rx} = 0$ corresponds to a person walking with the same velocity as that of the rain on the back.

The vertical rain velocity is defined as $v_{Ry}$; the way until a sheltered area is reached is called $s$. The angle of the rain to the floor in the moving coordinate system can easily be calculated:

$$\alpha = \arctan(v_{Ry}/v_{Rx}) \qquad \text{(figure 1, right panel)}. \tag{1}$$

Then the wetted plane $p$ (surrounded plane in figure 1, right panel) is defined as the projection of the person's shape (i.e. of the cylinder) onto the plane perpendicular to the rain in the moving system and has an area given by

$$p = \pi \cdot r^2 \cdot |\sin\alpha| + h \cdot 2r \cdot \cos\alpha, \tag{2}$$

where the first term is the area of the projection of the cylinder's base and the second is that of the rectangle which is the cylinder's cross-section.

The complete wetting $w$ must be proportional to the wetted plane, to the velocity of the rain with respect to this plane, and to the time of getting wet:

$$w \propto p \cdot \sqrt{v_{Rx}^2 + v_{Ry}^2} \cdot s/v_x = (\pi \cdot r^2 \cdot |\sin\alpha| + h \cdot 2r \cdot \cos\alpha) \cdot \sqrt{v_{Rx}^2 + v_{Ry}^2} \cdot s/v_x. \tag{3}$$

With $\sin(\arctan x) = \frac{x}{\sqrt{1+x^2}}$ and $\cos(\arctan x) = \frac{1}{\sqrt{1+x^2}}$, equation (3) can be reduced to

$$w \propto \left(\pi \cdot r^2 \frac{v_{Ry}/|v_{Rx}|}{\sqrt{1 + (v_{Ry}/v_{Rx})^2}} + 2hr \frac{1}{\sqrt{1 + (v_{Ry}/v_{Rx})^2}}\right) \cdot \sqrt{v_{Rx}^2 + v_{Ry}^2} \cdot \frac{s}{v_x} \tag{4}$$

$$= \pi \cdot r^2 \cdot s \cdot \frac{v_{Ry}}{v_x} + 2hr \cdot s \cdot \frac{|v_H - v_x|}{v_x}. \tag{5}$$

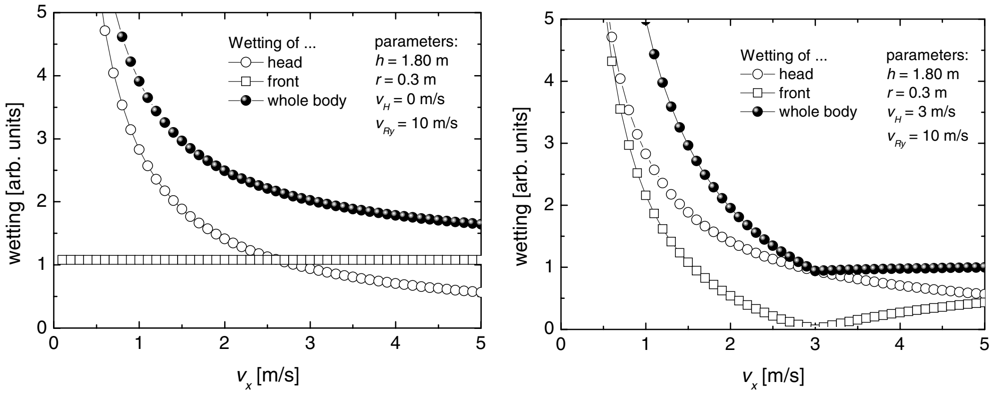


**Figure 2.** Wetting of head/front/the whole body, dependent on the walking velocity $v_x$. Left panel: $v_H = 0$, right panel: $v_H = 3\ \mathrm{m\,s^{-1}}$.

The first part of equation (5) describes the rain on the head of the person; the second part calculates the rain on front or back, respectively. Both parts depend on the velocity $v_x$ of the person.

Rewriting equation (5) leads to

$$w \propto \begin{cases} \dfrac{A_H v_{Ry} + A_v v_H}{v_x} - A_v (v_x < v_H) \\ \dfrac{A_H v_{Ry} - A_v v_H}{v_x} + A_v (v_x > v_H) \end{cases} \tag{6}$$

where $A_H$ is the area of the horizontal plane and $A_v$ is the area of the vertical plane (front of the person). Although derived in a different way, this formula is identical to the results of De Angelis (who does not give any derivation) [2] and Bailey [4]. Moreover, if the result is written in this way, it is easier to recognize that the graph of wetness $w$ consists of two hyperbolas. While in the first case ($v_x < v_H$), the first term is always positive, in the second case the numerator can be positive or negative. Now it is easy to see that for $A_H v_{Ry} - A_v v_H < 0$, the graph slopes upward, giving a minimum in $w$ for $v_x = v_H$, while for the numerator being positive there is no optimum velocity.

In the next section, the influence of different parameters on the solution for the minimization of the wetting $w$ will be discussed.

## 3. Results

The influence of the different variables in the formula derived above can easily be tested with a computer program such as Microsoft® Excel or Microcal Origin®, allowing the students to 'play' with the formula and to get a feeling for the parameters.

In figure 2, both terms of formula (5) and the whole wetting are depicted for different horizontal components $v_H$ of the rain. The rain always comes from behind. The range of $v_x$ is chosen for an average person who has to run a longer distance—while sprinters can run at a high speed of about 10 m s$^{-1}$ for 100 m, more than half of this value does not seem reasonable for someone running a longer distance in normal clothes without much training. For the other parameters, reasonable values are chosen too.

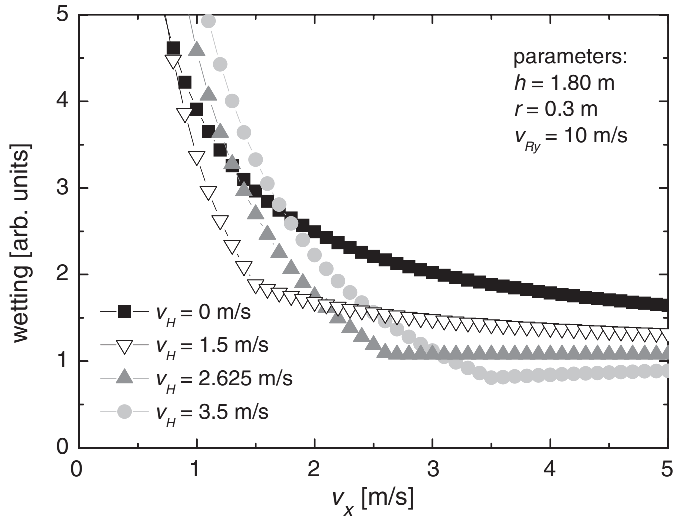


**Figure 3.** Wetting, calculated for the whole body (head + front), depending on the walking velocity $v_x$, for different horizontal rain velocities from 'back' (i.e. in the walking direction). An optimal walking velocity (minimum of wetting) below the maximum speed may exist, depending on the parameter set.

For $v_H = 0$ (left panel), the wetting of the head becomes smaller with larger walking velocity $v_x$, while the wetting of the front is constant. In this case apparently running at top speed is the best choice.

For $v_H = 3$ m s$^{-1}$ (right panel), however, this behaviour changes. While the wetting of the head is identical to $v_H = 0$ (compare equation (5) in which the first part is independent of $v_H$), the wetting of the front now shows a qualitatively different behaviour. If the person runs at the speed of rain from behind ($v_x = 3$ m s$^{-1}$), the wetting of the front vanishes, leading to a wetting minimum. For higher velocities, a limiting value is approached. With the chosen parameters, the person should run slowly at the horizontal speed of the rain. It should be emphasized here in this case that the faster the person runs, the wetter he or she gets—which is contrary to most other results.

Figure 2 also shows that for rain without horizontal component, the wetting of the front is independent of the walking speed (horizontal line in left panel).

In order to examine the dependence of the wetting minimum on the horizontal rain velocity $v_H$, the wetting of the whole body (head and front/back) is shown in figure 3 for a series of $v_H$. Obviously, a minimum occurs if $v_H > v_{Ry} A_H / A_v \approx 2.625$ m s$^{-1}$ for the chosen parameters, with a horizontal line of constant wetness for $v_H = v_{Ry} A_H / A_v$.

Compared to the results of De Angelis for purely vertical rain [2], figure 3 shows that more than the 10% reduction of the number of raindrops claimed in his paper is possible by running at the ideal speed instead of walking at a typical speed of about 1 m s$^{-1}$. Instead, for rain without horizontal component, the wetness at fast running velocities is about 50% less than that at normal walking speed, which agrees well with the experimental results in [3].

But what happens if the person walking in the rain is not as slim as assumed before? Figure 4 shows the influence of the diameter of the cylinder illustrating the running person. The horizontal rain velocity is identical to the one in figure 2 (right panel).

While for slimmer people (lower graphs) there exists a minimum for running at the speed of the rain on their back, heavier people should run as fast as possible since the minimum

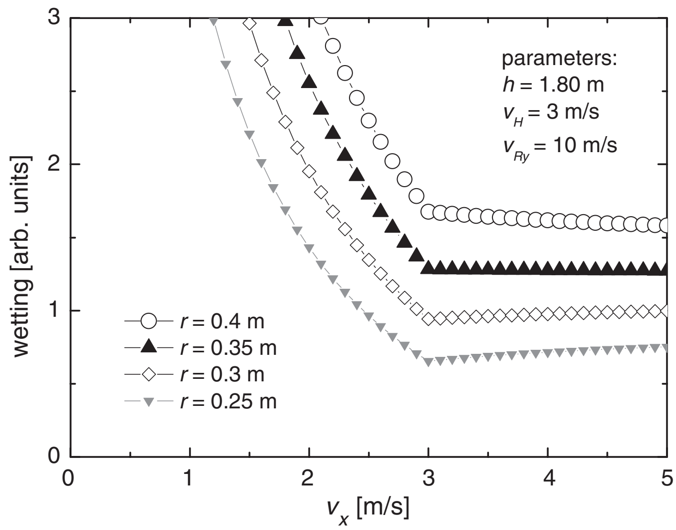


**Figure 4.** Wetting, calculated for the whole body (head + front/back), depending on the walking velocity $v_x$, for different radii of the 'walking cylinder', for $v_H = 3$ ms$^{-1}$ (rain from behind, i.e. in the walking direction).

vanishes. This behaviour can easily be explained—while the wetting of the head is proportional to $r^2$, the wetted front plane is proportional to $r$, leading to a stronger influence of the front part for less heavy people and thus a relatively more pronounced minimum for running with the speed of the rain on the back.

The distinction between both possibilities depends on the choice of the other parameters, as can clearly be seen in figure 3 for different horizontal rain velocities $v_H$. Obviously, higher values for $v_{Ry}$ lead to a larger influence of the head part of the formula derived above and thus to a less pronounced (or even vanishing) minimum for running at the speed of the rain; while for taller people (higher values of $h$), the front/back part of the formula becomes more important.

The calculations can also be performed for rain on the front of a person. In this case, independent of the horizontal speed of the rain and the dimensions of the walking person, running at maximum speed turns out to be the best solution.

## 4. Further problems

While the results depicted above already help the students to get a feeling for the influence of the parameters under investigation, several questions still remain unanswered.

Since the dimensions of the walking person do influence the existence of an optimum speed, the geometrical description of this person can be optimized. For a cuboid body with the same diameter, for example, the wetted front plane remains the same, while the wetted head area becomes larger. Thus the results are shifted towards a stronger influence of the head part (see figure 2). But how will the result change if the body of a person is modelled as a sphere upon a cylinder? What if a water resistant hat is worn? What about a body with elliptical cross-section? What happens if arms and legs are simulated as separate cylinders? And what if they are additionally calculated with an average angle in relation to the rest of the body in order to include the movements of the extremities while running, or if the whole body changes its angle relative to the ground?

In addition to these simple geometric changes in the formula derived above, the influence of other effects can be estimated by the students. The rain can come from the side or in gusts (i.e. with alternating speed) and can have a different vertical velocity component. A wetting probability depending on the velocity of the rain drops relative to the person's clothes or the angle of incidence can be implemented, and water splashing up from the ground against the legs of the running person can be included, to name just a few. Several questions are awaiting answers, only limited by the students' imagination.

## 5. Conclusion

The question of whether to walk or to run in the rain has been answered by a simple intuitive model which allows for numerous modifications. The idea of Bailey [4] that the ideal solution depends on the dimensions of the person could be supported not only if rain coming from the side is taken into consideration. The reason for this behaviour has been illustrated by comparing the influence of the head and body on the whole wetting. It has been clearly demonstrated that if a wetting minimum exists, running at top speed is not the best option to take.

As we have shown with a small selection of open questions, the apparently simple problem of a person in the rain is not only capable of training students to solve problems by means of typical programs for data evaluation, it also allows for a number of more or less complex variations of our calculation, offering students a broad field to 'play' with parameters in order to test their influence and the plausibility of the complete model.

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